

12.2 | Magnetic Field Due to a Thin Straight Wire

Learning Objectives

By the end of this section, you will be able to:

- Explain how the Biot-Savart law is used to determine the magnetic field due to a thin, straight wire.
- Determine the dependence of the magnetic field from a thin, straight wire based on the distance from it and the current flowing in the wire.
- Sketch the magnetic field created from a thin, straight wire by using the second right-hand rule.

How much current is needed to produce a significant magnetic field, perhaps as strong as Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted in Chapter 28 that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire? We can use the Biot-Savart law to answer all of these questions, including determining the magnetic field of a long straight wire.

Figure 12.5 shows a section of an infinitely long, straight wire that carries a current I . What is the magnetic field at a point P , located a distance R from the wire?

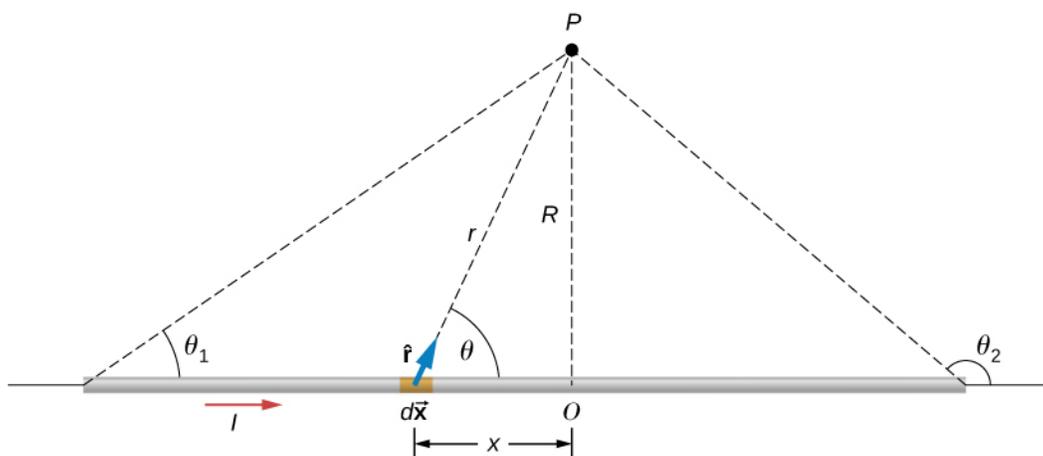


Figure 12.5 A section of a thin, straight current-carrying wire. The independent variable θ has the limits θ_1 and θ_2 .

Let's begin by considering the magnetic field due to the current element $I d\vec{x}$ located at the position x . Using the right-hand rule 1 from the previous chapter, $d\vec{x} \times \hat{\mathbf{r}}$ points out of the page for any element along the wire. At point P , therefore, the magnetic fields due to all current elements have the same direction. This means that we can calculate the net field there by evaluating the scalar sum of the contributions of the elements. With $|d\vec{x} \times \hat{\mathbf{r}}| = (dx)(1)\sin\theta$, we have from the Biot-Savart law

$$B = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I \sin\theta dx}{r^2}. \quad (12.5)$$

The wire is symmetrical about point O , so we can set the limits of the integration from zero to infinity and double the answer, rather than integrate from negative infinity to positive infinity. Based on the picture and geometry, we can write expressions for r and $\sin\theta$ in terms of x and R , namely:

$$r = \sqrt{x^2 + R^2}$$

$$\sin \theta = \frac{R}{\sqrt{x^2 + R^2}}$$

Substituting these expressions into **Equation 12.5**, the magnetic field integration becomes

$$B = \frac{\mu_0 I}{2\pi} \int_0^\infty \frac{R dx}{(x^2 + R^2)^{3/2}} \quad (12.6)$$

Evaluating the integral yields

$$B = \frac{\mu_0 I}{2\pi R} \left[\frac{x}{(x^2 + R^2)^{1/2}} \right]_0^\infty \quad (12.7)$$

Substituting the limits gives us the solution

$$B = \frac{\mu_0 I}{2\pi R} \quad (12.8)$$

The magnetic field lines of the infinite wire are circular and centered at the wire (**Figure 12.6**), and they are identical in every plane perpendicular to the wire. Since the field decreases with distance from the wire, the spacing of the field lines must increase correspondingly with distance. The direction of this magnetic field may be found with a second form of the right-hand rule (illustrated in **Figure 12.6**). If you hold the wire with your right hand so that your thumb points along the current, then your fingers wrap around the wire in the same sense as \vec{B} .

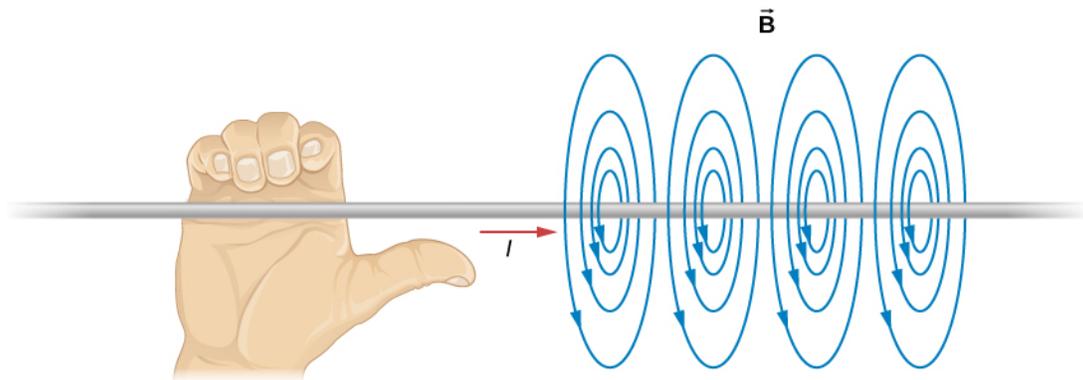


Figure 12.6 Some magnetic field lines of an infinite wire. The direction of \vec{B} can be found with a form of the right-hand rule.

The direction of the field lines can be observed experimentally by placing several small compass needles on a circle near the wire, as illustrated in **Figure 12.7**. When there is no current in the wire, the needles align with Earth's magnetic field. However, when a large current is sent through the wire, the compass needles all point tangent to the circle. Iron filings sprinkled on a horizontal surface also delineate the field lines, as shown in **Figure 12.7**.

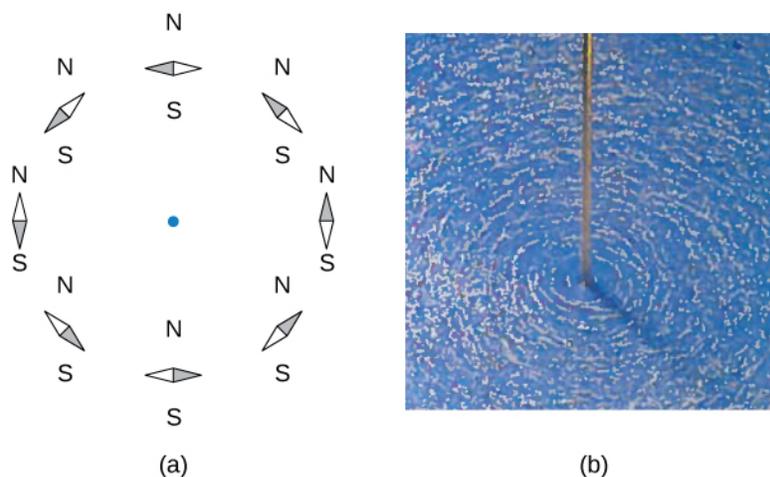


Figure 12.7 The shape of the magnetic field lines of a long wire can be seen using (a) small compass needles and (b) iron filings.

Example 12.3

Calculating Magnetic Field Due to Three Wires

Three wires sit at the corners of a square, all carrying currents of 2 amps into the page as shown in **Figure 12.8**. Calculate the magnitude of the magnetic field at the other corner of the square, point P , if the length of each side of the square is 1 cm.

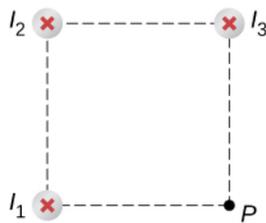


Figure 12.8 Three wires have current flowing into the page. The magnetic field is determined at the fourth corner of the square.

Strategy

The magnetic field due to each wire at the desired point is calculated. The diagonal distance is calculated using the Pythagorean theorem. Next, the direction of each magnetic field's contribution is determined by drawing a circle centered at the point of the wire and out toward the desired point. The direction of the magnetic field contribution from that wire is tangential to the curve. Lastly, working with these vectors, the resultant is calculated.

Solution

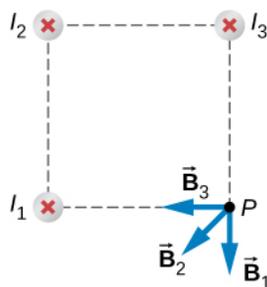
Wires 1 and 3 both have the same magnitude of magnetic field contribution at point P :

$$B_1 = B_3 = \frac{\mu_o I}{2\pi R} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2 \text{ A})}{2\pi(0.01 \text{ m})} = 4 \times 10^{-5} \text{ T.}$$

Wire 2 has a longer distance and a magnetic field contribution at point P of:

$$B_2 = \frac{\mu_o I}{2\pi R} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2 \text{ A})}{2\pi(0.01414 \text{ m})} = 3 \times 10^{-5} \text{ T.}$$

The vectors for each of these magnetic field contributions are shown.



The magnetic field in the x -direction has contributions from wire 3 and the x -component of wire 2:

$$B_{\text{net } x} = -4 \times 10^{-5} \text{ T} - 2.83 \times 10^{-5} \text{ T} \cos(45^\circ) = -6 \times 10^{-5} \text{ T}.$$

The y -component is similarly the contributions from wire 1 and the y -component of wire 2:

$$B_{\text{net } y} = -4 \times 10^{-5} \text{ T} - 2.83 \times 10^{-5} \text{ T} \sin(45^\circ) = -6 \times 10^{-5} \text{ T}.$$

Therefore, the net magnetic field is the resultant of these two components:

$$\begin{aligned} B_{\text{net}} &= \sqrt{B_{\text{net } x}^2 + B_{\text{net } y}^2} \\ B_{\text{net}} &= \sqrt{(-6 \times 10^{-5} \text{ T})^2 + (-6 \times 10^{-5} \text{ T})^2} \\ B_{\text{net}} &= 8 \times 10^{-5} \text{ T}. \end{aligned}$$

Significance

The geometry in this problem results in the magnetic field contributions in the x - and y -directions having the same magnitude. This is not necessarily the case if the currents were different values or if the wires were located in different positions. Regardless of the numerical results, working on the components of the vectors will yield the resulting magnetic field at the point in need.



12.3 Check Your Understanding Using **Example 12.3**, keeping the currents the same in wires 1 and 3, what should the current be in wire 2 to counteract the magnetic fields from wires 1 and 3 so that there is no net magnetic field at point P?

12.3 | Magnetic Force between Two Parallel Currents

Learning Objectives

By the end of this section, you will be able to:

- Explain how parallel wires carrying currents can attract or repel each other
- Define the ampere and describe how it is related to current-carrying wires
- Calculate the force of attraction or repulsion between two current-carrying wires

You might expect that two current-carrying wires generate significant forces between them, since ordinary currents produce magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to define the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long, straight, and parallel conductors separated by a distance r can be found by applying what we have developed in the preceding sections. **Figure 12.9** shows the wires, their currents, the field created by one wire, and the consequent force the other wire experiences from the created field. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is